Lecture 31

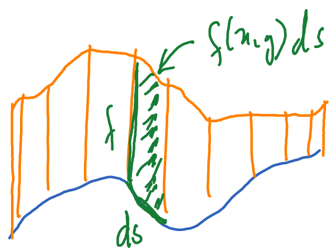
Tuesday, March 22, 2022 11:24 PM

* Prayer
* Spiritual thought

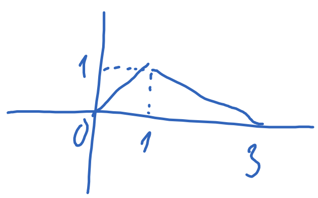
Line integral
$$\int_C f(x,y,\dots)\,ds = \int_a^b f(x(t), y(t), \dots)\,|r'(t)|\,dt$$

$$C:\ r(t) = (x(t), y(t), \dots), \quad a \le t \le b.$$

Applications: mass of a wire, area of a wall built on a curve.

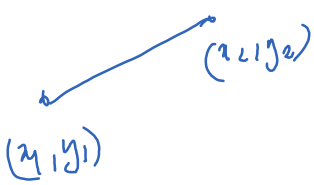


Ex
$$\int_C xy\,ds = \int_{C_1} xy\,ds + \int_{C_2} xy\,ds$$



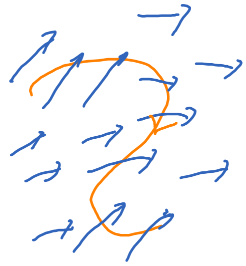
$$C_1: \begin{cases} x = t \\ y = t \end{cases} \quad 0 \le t \le 1$$

$$C_2: \begin{cases} x = 1 + 2t \\ y = 1 - t \end{cases} \quad 0 \le t \le 1$$



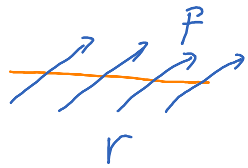
$$\begin{cases} x = x_1 + t(x_2 - x_1) \\ y = y_1 + t(y_2 - y_1) \end{cases} \quad 0 \le t \le 1$$

* Line integral of a vector field:



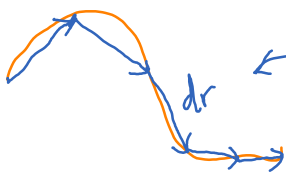
An object moving inside a force field.

What is the work done by the force?



$\text{work} = |F||r|\cos\alpha$

$= F \cdot r$



work done on this segment is $F \cdot dr$

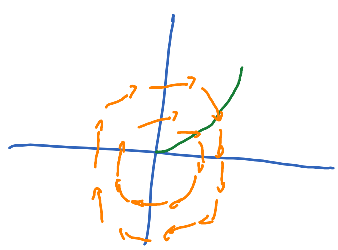
$$\text{Total work} = \int_C F \cdot dr$$

Ex $F(x,y) = (y, -x)$

[Imagine swimming under this current. Are you "helped" or "hindered" by the current?]

$C : r(t) = (t, t^2)$

$0 \le t \le 1$



$$\text{work} = \int_C F \cdot dr$$

$$dr = (dt, 2t\,dt) = (1, 2t)\,dt$$

$$F = (y, -x) = (t^2, -t)$$

$$\int_C F \cdot dr = \int_0^1 (t^2, -t) \cdot (1, 2t)\,dt = \int_0^1 (t^2 - 2t^2)\,dt = -\frac{t^3}{3}\Big|_0^1 = -\frac{1}{3}$$

Special vector field: Conservative vector field

$$F = \nabla \phi$$

Conservative vector field

potential function

Line integral of a conservative vector field only depends on the endpoints of the curve, not anything in the middle of the curve.